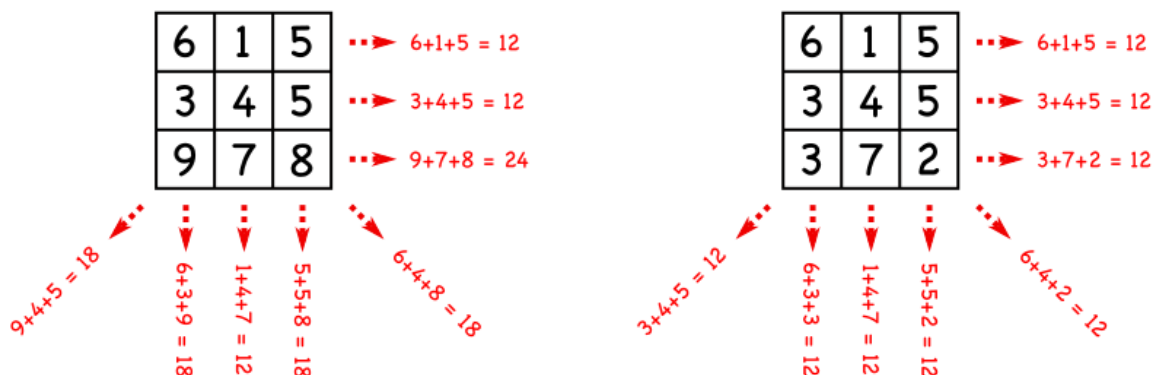


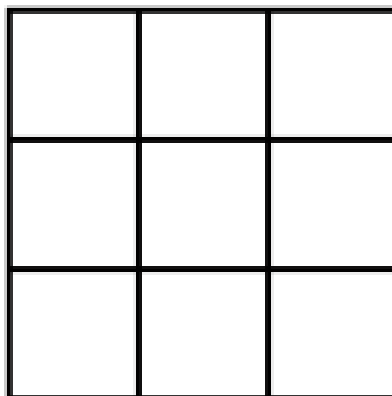
Puzzle of the Week

Magic Squares – 3

In a *Magic Square*, all the rows, columns and diagonals add up to the same number. This first square is not a Magic Square. The second one is a Magic Square with a constant sum of 12.



THE CHALLENGE: Use each of the numbers from 0 to 8 once to complete this Magic Square.



0 1 2 3 4 5 6 7 8

EXPLORATION: Can you find more than one way to do it? What do the different ways have in common? How would your answer change if you used the numbers from 1 to 9? How about the even numbers from 2 to 18?

Puzzle of the Week

Magic Squares – 3 – Notes

THE CHALLENGE: Let your students play with this. If they pay attention to what they're doing, they'll discover interesting relationships and get a lot out of it. For young students, there is absolutely no need to go into any kind of careful analysis. What follows is a more analytical way to find the solutions.

Common Sum: The simplest way to start analyzing this puzzle is to find the common sum. Each row adds up to the common sum. Also, the three rows contain the numbers from 0 to 8 and add up to three times the common sum. Therefore, three times the common sum is 36 (the sum of 0 to 8), so the common sum is 12.

Central Square: The next step is to add up the four lines that go through the center square. The common sum is 12 and there are four lines, so their sum must be $4 \times 12 = 48$. Alternatively, the four lines contain every number once, plus the central square three more times. The sum of the numbers from 0 to 8 is 36. So, 48 equals 36 plus 3 times the central square. So, the central square must be 4.

Adding up to 12: There are surprisingly few ways to add up to 12. They are:

(0 4 8) (1 4 7) (2 4 6) (3 4 5) (0 5 7) (1 3 8) (1 5 6) (2 3 7)

You can figure out a lot for this list. Look at how often a number appears in a triplet:

- 4 times: 4
- 3 times: 1, 3, 5, 7
- 2 times: 0, 2, 6, 8

Next, compare this to how many times a square in the diagram is in one of the lines. You'll see that the center square is in four lines, the corner squares are in three lines, and the middle of the sides are in two lines. This is another way to see that the center square must be 4. Also, the corners must be 1, 3, 5, and 7, and the middle of the sides must be 0, 2, 6, and 8.

Fill up the Square: The hard work is done. Start with 4 in the middle and put 7 in one corner. Note that 0 must go next to the 7 on one side or the other (otherwise the 8 would be forced next to the 7). You will have no choices after that. One answer, by rows, is: (7 0 5) (2 4 6) (3 8 1). Notice that this is the same as any other answer by rotating the square and possibly flipping it.

EXPLORATION: As noted in the last paragraph, all the solutions are essentially the same - rotate the square until the 7's are in the same corner, and take the mirror image (if needed) along the diagonal to put the 0 in the same position.

Solving this puzzle for 1 to 9 would mean adding 1 to every entry in the 0 to 8 solution. Solving this puzzle for 2 to 18 would mean doubling all the entries for the 1 to 9 solution.